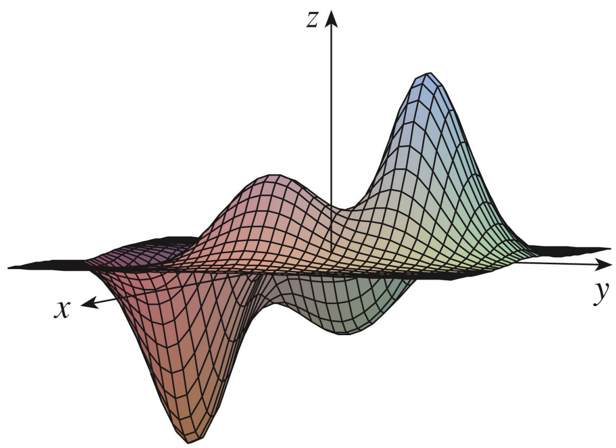
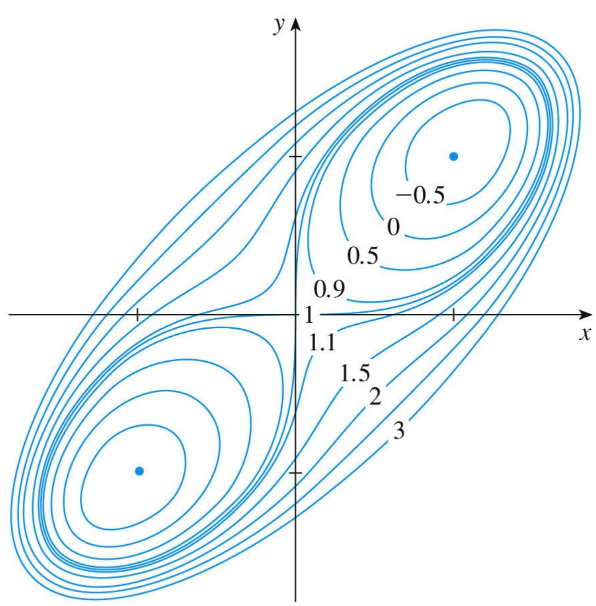
Lesson 19. Local Minima and Maxima

1 Local minima and maxima

- Let f be a function of two variables
- f has a **local maximum** at (a, b) if $f(a, b) \geq f(x, y)$ for all (x, y) "close" to (a, b)
- f has a **local minimum** at (a, b) if $f(a, b) \leq f(x, y)$ for all (x, y) "close" to (a, b)



Example 1. The contour map for $f(x, y) = x^4 + y^4 - 4xy + 1$ is shown below. Find the local maxima and minima of f.



2 Critical points: how to find local minima and maxima

- (a, b) is a **critical point** of f if

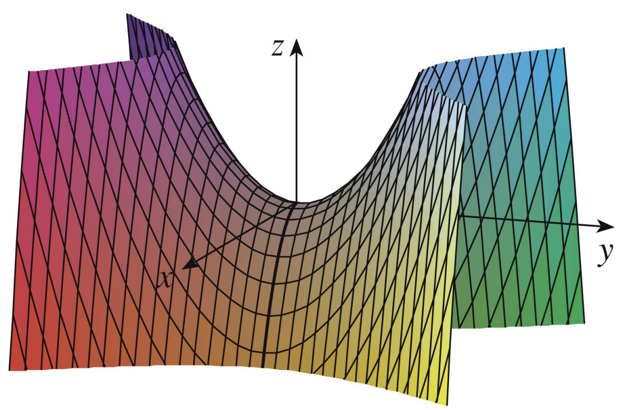
  or if one of these partial derivatives does not exist

- If f has a local minimum or maximum at (a, b), then (a, b) is a critical point

- Finding local minima and maxima of f:

  1. Find all critical points of f
  2. Categorize each critical point using the **second derivatives test**:
     - Let $D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$
     - If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a
     - If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a
     - If $D(a, b) < 0$, then (a, b) is a of f
     - If $D(a, b) = 0$, the test gives no information

- Saddle points
  - Highest point in one direction, lowest point in the other direction
  - Graphically:



  - Saddle points look like hyperbolas in contour maps (see $(0, 0)$ in Example 1)

Example 2. Find the local minimum and maximum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

Example 3. Find the local minimum and maximum values and saddle points of $f(x, y) = y(e^x - 1)$.